AB Calculus Section 1.4

One sided limits/Continuity and Intermediate Value Theorem

Name _____

1. For the function F graphed in the accompanying figure, find
   (a) $\lim_{x \to -2^-} F(x)$ (b) $\lim_{x \to -2^+} F(x)$
   (c) $\lim_{x \to -2} F(x)$ (d) $F(-2)$.

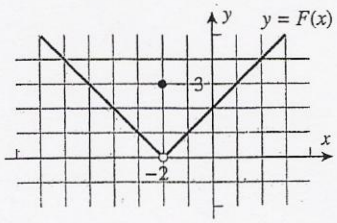


Figure Ex-1

2. For the function ϕ graphed in the accompanying figure, find
   (a) $\lim_{x \to 4^-} \phi(x)$ (b) $\lim_{x \to 4^+} \phi(x)$
   (c) $\lim_{x \to 4} \phi(x)$ (d) $\phi(4)$.

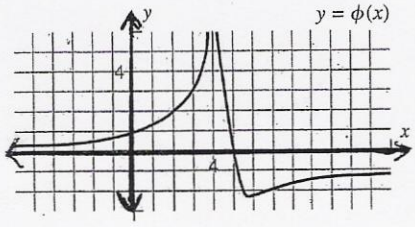


Figure Ex-2

Find the limit (if it exists).

3. Let

$$f(x) = \begin{cases} x - 1, & x \le 3 \\ 3x - 7, & x > 3 \end{cases}$$

Find
(a) $\lim_{x \to 3^-} f(x)$

4. Let

$$g(t) = \begin{cases} t^2, & t \ge 0 \\ t - 2, & t < 0 \end{cases}$$

Find
(a) $\lim_{t \to 0^-} g(t)$ (b) $\lim_{t \to 0} g(t)$.

5. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12 - 2x}{3}, & x > 3 \end{cases}$

6. $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

7. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

8. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$

9. If possible, choose k so that the following function is continuous on any interval:

$$f(x) = \begin{cases} \dfrac{5x^3 - 10x^2}{x - 2} & x \ne 2 \\ k & x = 2. \end{cases}$$

10. Find a value of the constant k, if possible, that will make the function continuous everywhere.

(a) $f(x) = \begin{cases} 7x - 2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$

(b) $f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x + k, & x > 2 \end{cases}$

In Exercises 11–14, show that there is a number c, with $0 \le c \le 1$, such that $f(c) = 0$.

11. $f(x) = x^3 + x^2 - 1$

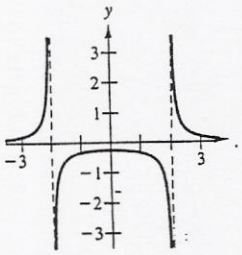
12. $f(x) = e^x - 3x$

13. $f(x) = x - \cos x$

14. $f(x) = 2^x - 1/x$

In Exercises 23–28 find the x-values (if any) at which f is not continuous.

23. $f(x) = \dfrac{1}{x^2 - 4}$



24. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$

25. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

26. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \ge 1 \end{cases}$

27. $f(x) = \begin{cases} \dfrac{x}{2} + 1, & x < 2 \\ 3 - x, & x > 2 \end{cases}$

28. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

AB Calculus Section 1.4

One sided limits/Continuity and Intermediate Value Theorem

Name Key

1. For the function F graphed in the accompanying figure, find
(a) $\lim_{x \to -2^-} F(x) = 0$ (b) $\lim_{x \to -2^+} F(x) = 0$
(c) $\lim_{x \to -2} F(x) = 0$ (d) $F(-2). = 3$

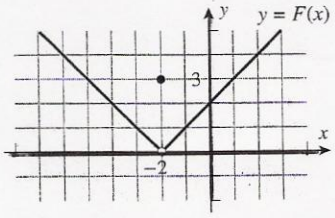


Figure Ex-1

2. For the function ϕ graphed in the accompanying figure, find
(a) $\lim_{x \to 4^-} \phi(x) = +\infty$ (b) $\lim_{x \to 4^+} \phi(x) = +\infty$
(c) $\lim_{x \to 4} \phi(x)$ DNE (d) $\phi(4)$. DNE

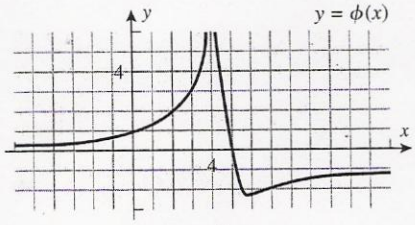


Figure Ex-2

Find the limit (if it exists).

3. Let
$$f(x) = \begin{cases} x - 1, & x \le 3 \\ 3x - 7, & x > 3 \end{cases}$$
Find
(a) $\lim_{x \to 3^-} f(x) = 2$

4. Let
$$g(t) = \begin{cases} t^2, & t \ge 0 \\ t - 2, & t < 0 \end{cases}$$
Find
(a) $\lim_{t \to 0^-} g(t) = -2$ (b) $\lim_{t \to 0} g(t)$. DNE

5. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} \frac{x+2}{2}, & x \le 3 \\ \frac{12 - 2x}{3}, & x > 3 \end{cases}$ DNE

6. $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$ = 2

7. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$ = 2

8. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$ DNE

9. If possible, choose k so that the following function is continuous on any interval:
$$f(x) = \begin{cases} \frac{5x^3 - 10x^2}{x - 2} & x \ne 2 \\ k & x = 2. \end{cases}$$
K = 20

10. Find a value of the constant k, if possible, that will make the function continuous everywhere.
(a) $f(x) = \begin{cases} 7x - 2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$ K = 5
(b) $f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x + k, & x > 2 \end{cases}$ K = 4/3

In Exercises 11–14, show that there is a number c, with $0 \le c \le 1$, such that $f(c) = 0$.

11. $f(x) = x^3 + x^2 - 1$ — 1) polynomial is continuous 2) $f(0) \ne f(1)$, $-1 \ne 1$

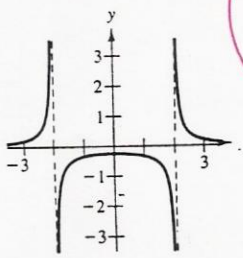
12. $f(x) = e^x - 3x$ — 1) polyn. is continuous 2) $f(0) \ne f(1)$; $1 \ne e - 3$

13. $f(x) = x - \cos x$ — Is $f(x)$ cont? 1) $f(c)$'s are defined ($y = x$ is cont. & $\cos x$ is cont.) 2) $f(0) \ne f(1)$, $0 - 1 \ne 1 - \cos 1$

14. $f(x) = 2^x - 1/x$

In Exercises 23–28 find the x-values (if any) at which f is not continuous.

23. $f(x) = \dfrac{1}{x^2 - 4}$



x = 2, x = −2

24. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$ x = 1

25. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$ none

26. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \ge 1 \end{cases}$ none

27. $f(x) = \begin{cases} \frac{x}{2} + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$ none

28. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$ x = 2