Many artists use representations of three-dimensional figures in their sculptures, such as those seen here in downtown Los Angeles.

Pershing Square, Los Angeles
Vocabulary

Choose the best term from the list to complete each sentence.

1. A(n) __?__ is a number written as a ratio that represents a part of a whole.
2. A(n) __?__ is another way of writing a fraction.
3. To multiply 7 by the fraction \(\frac{2}{3}\), multiply 7 by the __?__ of the fraction and then divide the result by the __?__ of the fraction.

Complete these exercises to review skills you will need for this chapter.

Find the Square of a Number

Evaluate.

4. \(16^2\)  
5. \(9^2\)  
6. \((4.1)^2\)  
7. \((0.5)^2\)
8. \(\left(\frac{1}{4}\right)^2\)  
9. \(\left(\frac{2}{5}\right)^2\)  
10. \(\left(\frac{1}{2}\right)^2\)  
11. \(\left(\frac{2}{3}\right)^2\)

Multiply Fractions

Multiply.

12. \(\frac{1}{2} \times \frac{8}{9}\)  
13. \(\frac{1}{2} \times \frac{2}{3}\)  
14. \(\frac{1}{3} \times \frac{9}{11}\)  
15. \(\frac{1}{3} \times \frac{6}{7}\)
16. \(\frac{2}{5} \times \frac{5}{4}\)  
17. \(\frac{1}{5} \times \frac{4}{3}\)  
18. \(\frac{2}{3} \times \frac{6}{11}\)  
19. \(\frac{3}{4} \times \frac{5}{6}\)

Decimal Operations

Multiply. Write each answer to the nearest tenth.

20. \(3.14 \times 2.5\)  
21. \(3.14 \times 1.25\)  
22. \(3.14 \times 3.5\)  
23. \(3.14 \times 1.75\)

Multiply with Fractions and Decimals

Multiply. Write each answer to the nearest tenth.

24. \(3.14 \times 7\)  
25. \(3.14 \times 10\)
26. \(3.14 \times 20\)  
27. \(3.14 \times 15\)
28. \(\frac{22}{7} \times 14\)  
29. \(\frac{22}{7} \times 21\)
30. \(1\frac{1}{3} \times 15\)  
31. \(1\frac{1}{3} \times 36\)
The information below “unpacks” the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

<table>
<thead>
<tr>
<th>California Standard</th>
<th>Academic Vocabulary</th>
<th>Chapter Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths). (Lab 10-2)</td>
<td>e.g. abbreviation that stands for the Latin phrase <em>exempli gratia</em>, which means “for example” fixed in this case, set or unchanging</td>
<td>You analyze the relationship between length and volume in three-dimensional figures.</td>
</tr>
<tr>
<td>MG2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor. (Lab 10-2; Lessons 10-4, 10-7)</td>
<td>compute determine by using mathematical operations such as addition and multiplication dimensions measurement in length, width, or thickness</td>
<td>You find the surface area and volume of similar three-dimensional figures.</td>
</tr>
<tr>
<td>MG2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft.²] = [144 in.²]; 1 cubic inch is approximately 16.38 cubic centimeters or [1 in.³] = [16.38 cm³]). (Lesson 10-2)</td>
<td>relate to show a connection between approximately estimated to be</td>
<td>You understand the measurements used in describing the area and volume of objects. Example: Area is measured in square units, such as square inches (in²) and square meters (m²). Volume is measured in cubic units, such as cubic inches (in³) and cubic meters (m³).</td>
</tr>
<tr>
<td>MG3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones. (Labs 10-4, 10-5)</td>
<td>construct to build or draw</td>
<td>You make nets for cylinders, prisms, and cones.</td>
</tr>
</tbody>
</table>

Standards MG2.1 and MG2.2 are also covered in this chapter. To see these standards unpacked, go to Chapter 9, p. 432.
Writing Strategy: Draw Three-Dimensional Figures

When you encounter a three-dimensional figure such as a cylinder, cone, sphere, prism, or pyramid, it may help you to make a quick sketch so that you can visualize its shape. Use these tips to help you draw quick sketches of three-dimensional figures.

- **Cylinder**: Draw two ellipses, one above the other, as shown. Make half of the lower one dashed. Draw two segments connecting the ellipses.

- **Prism**: Draw two parallelograms, one above the other. Make two sides of the lower one dashed. Draw segments connecting the vertices of the parallelograms. Use a dashed segment for the hidden edge.

- **Sphere**: Draw a circle and its center. Draw an ellipse inside the circle. Make the top half of the ellipse dashed.

- **Cone**: Draw an ellipse and a point above it. Make the top half of the ellipse dashed. Draw two segments connecting the point to the ellipse.

- **Pyramid**: Draw a parallelogram and a point above it. Make two sides of the parallelogram dashed. Draw segments connecting the vertices of the parallelogram to the point. Use a dashed segment for the hidden edge.

**Try This**

1. Explain and show how to draw a cube, which is a prism with equal length, width, and height.
2. Draw a prism, starting with two hexagons. (Hint: Draw the hexagons as if you were viewing them at an angle.)
3. Draw a pyramid, starting with a triangle and a point above the triangle.
Explore Three-Dimensional Figures

A prism is a three-dimensional figure with two parallel and congruent polygons called the bases. The remaining edges join corresponding vertices of the bases so that the remaining surfaces are rectangles. You can use this definition to make models of prisms.

Activity 1

1. Use a ruler and protractor to draw two squares with side lengths of 3 centimeters on cardboard. Then cut out the squares.

2. From heavy paper, cut out a rectangle that measures 12 centimeters by 6 centimeters. Fold the paper into fourths as shown.

3. Fold the rectangle into an open-ended box, and tape together the 6-centimeter edges. Form a prism by taping the edges of the squares to the open ends of the box.

4. Use a ruler and protractor to draw two equilateral triangles with side lengths of 4 centimeters on cardboard. Then cut out the triangles. (Hint: Each angle of an equilateral triangle measures 60°.)

5. From heavy paper, cut out a rectangle that measures 12 centimeters by 6 centimeters. Fold the paper into thirds as shown.

6. Tape together the 6-centimeter edges. Form a prism by taping the edges of the triangles to the open ends of the folded paper.

Think and Discuss

1. What shape are the bases of each prism that you modeled?
2. What shape are the other surfaces of each prism?
3. How is the second prism that you modeled different from the first prism? How are the prisms alike?
Tell whether each figure below is a prism. Explain your answer.

1. 2. 3.

A cylinder is a three-dimensional figure with two parallel congruent circular bases. The third surface of a cylinder consists of all parallel circles of the same radius whose centers lie on the line segment joining the centers of the bases. You can use this definition to model a cylinder.

Activity 2

1. Use a compass to draw at least 10 circles with a radius of 3 centimeters each on cardboard. Then cut out the circles.

2. Poke a hole through the center of each circle.

3. Unbend a paper clip part way to form a right angle. Then push the paper clip through the center of each circle to model a cylinder. Use the paper clip to keep the stack of cardboard circles aligned.

Think and Discuss

1. Describe the bases of your cylinder.

2. How is your model of a cylinder different from your models of prisms? How are they the same?

Try This

Tell whether each figure below is a cylinder. Explain your answer.

1. 2. 3.
Three-dimensional figures have three dimensions: length, width, and height. A flat surface of a three-dimensional figure is a **face**. An **edge** is where two faces meet.

A **polyhedron** is a three-dimensional figure whose faces are all polygons. A **vertex** of a polyhedron is a point where three or more edges meet. The face that is used to name a polyhedron is called a **base**.

### Vocabulary
- **face**
- **edge**
- **polyhedron**
- **vertex**
- **base**
- **prism**
- **pyramid**
- **cylinder**
- **cone**

### Why learn this?
You can name and describe three-dimensional shapes used in historical structures. (See Exercises 19–22.)

### Three-Dimensional Figures

<table>
<thead>
<tr>
<th>Prisms</th>
<th>Pyramids</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Two parallel congruent bases that are polygons</td>
<td>• One base that is a polygon</td>
</tr>
<tr>
<td>• Remaining faces are parallelograms</td>
<td>• Remaining faces are triangles</td>
</tr>
</tbody>
</table>

### Naming Prisms and Pyramids

**Describe the bases and faces of each figure. Then name the figure.**

**A**
- There are two rectangular bases.
- There are four other rectangular faces.
- The figure is a rectangular prism.

**B**
- There are two triangular bases.
- There are three rectangular faces.
- The figure is a triangular prism.

**C**
- There is one hexagonal base.
- There are six triangular faces.
- The figure is a hexagonal pyramid.

**Example 1**

The bottom face of a prism is not always one of its bases. For example, the bottom face of the triangular prism in Example 1B is not one of its triangular bases.
Other three-dimensional figures include cylinders and cones. These figures are not polyhedrons because they are not made of faces that are all polygons.

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Cone</th>
</tr>
</thead>
</table>
| • Two parallel congruent bases that are circles  
  • Bases connected by a curved surface | • One base that is a circle  
  • Curved surface that comes to a point |
| ![Cylinder Diagram] | ![Cone Diagram] |

You can use properties to classify three-dimensional figures.

**Think and Discuss**

1. **Explain** how to identify a prism or a pyramid.
2. **Compare and contrast** cylinders and prisms. How are they alike? How are they different?
3. **Compare and contrast** pyramids and cones. How are they alike? How are they different?
10-1 Exercises

GUIDED PRACTICE

Describe the bases and faces of each figure. Then name the figure.

1. 

2. 

3. 

Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

4. 

5. 

6. 

INDEPENDENT PRACTICE

Describe the bases and faces of each figure. Then name the figure.

7. 

8. 

9. 

Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

10. 

11. 

12. 

PRACTICE AND PROBLEM SOLVING

Identify the three-dimensional figure described.

13. two parallel, congruent square bases and four other polygonal faces

14. two parallel, congruent circular bases and one curved surface

15. one triangular base and three other triangular faces

16. one circular base and one curved surface

Name two examples of the three-dimensional figure described.

17. two parallel, congruent bases

18. one base
19. The structures in the photo at right are tombs of ancient Egyptian kings. No one knows exactly when the tombs were built, but some archaeologists think the first one might have been built around 2780 B.C.E. Name the shape of the ancient Egyptian structures.

20. The Parthenon was built around 440 B.C.E. by the ancient Greeks. Its purpose was to house a statue of Athena, the Greek goddess of wisdom. Describe the three-dimensional shapes you see in the structure.

21. The Leaning Tower of Pisa began to lean as it was being built. To keep the tower from falling over, the upper sections (floors) were built slightly off center so that the tower would curve away from the way it was leaning. What shape is each section of the tower?

22. **Challenge** The stainless steel structure at right, called the Unisphere, became the symbol of the New York World’s Fair of 1964–1965. A sphere is a three-dimensional figure with a surface made up of all the points that are the same distance from a given point. Explain why the structure is not a sphere.

### Spiral Standards Review

23. **Multiple Choice** Which figure has six rectangular faces?

- A. Rectangular prism
- B. Triangular prism
- C. Triangular pyramid
- D. Rectangular pyramid

24. **Multiple Choice** Which figure does NOT have two congruent bases?

- A. Cube
- B. Pyramid
- C. Prism
- D. Cylinder

Add. Write each answer in simplest form. (Lesson 2-3)

25. $\frac{2}{5} + \frac{3}{8}$  
26. $\frac{1}{16} + \frac{4}{9}$  
27. $\frac{7}{9} + \frac{11}{12}$  
28. $\frac{1}{10} + \frac{1}{16}$

29. A store sells two sizes of detergent: 300 ounces for $21.63 and 100 ounces for $6.99. Which size detergent has the lowest price per ounce? (Lesson 5-2)
Explore Volume of Prisms

You can use models to explore the volume of rectangular prisms.

**Activity 1**
Use five different-sized rectangular prisms, such as empty cartons.

a. Cover the bottom of each prism with cubes to find the area of the prism’s base. Record the information in a table.

b. Fill the prism with cubes. Find the height. Then count the cubes to find the prism’s volume. Record the information in a table.

**Think and Discuss**
1. What do you notice about the relationship between the area of the base, the height, and the volume of the rectangular prisms?

2. Make a conjecture about how to find the volume of any rectangular prism.

**Activity 2**
Consider several prisms with square bases of various side lengths and a fixed height of 5 units.

a. Find the area of each square base. Record the information in a table.

b. Use your conjecture from Activity 1 to find the volume of each prism. Record your answers in a table.

c. Make a graph by plotting the volume of each prism as a function of the side length of the base.

**Think and Discuss**
1. Think about the functions you studied in Chapter 7. Which type of function best matches the graph?

**Try This**
1. Consider cubes with edge lengths of 1 unit, 2 units, 3 units, 4 units, and 5 units. Make a graph by plotting the volume of each cube as a function of the edge length. Which type of function best matches the graph?
In Lesson 9-1, you saw that the area of a two-dimensional figure is the number of unit squares needed to cover the figure. Similarly, any three-dimensional figure can be filled completely with congruent cubes and parts of cubes. The volume of a three-dimensional figure is the number of cubes it can hold. Each cube represents a unit of measure called a cubic unit.

Finding the Volume of Prisms and Cylinders

**EXAMPLE 1**

Find the volume of the figure to the nearest tenth.

**Prism:** The volume $V$ of a prism is the area of the base $B$ times the height $h$.

**Cylinder:** The volume of a cylinder is the area of the base $B$ times the height $h$.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Rectangular prism</th>
<th>Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words</strong></td>
<td>$V = Bh$</td>
<td>$V = Bh$</td>
</tr>
<tr>
<td><strong>Formula</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Numbers</strong></td>
<td>$B = 5(2)$, $V = (10)(3)$</td>
<td>$B = \pi (2^2)$, $V = (4\pi)(6)$</td>
</tr>
<tr>
<td></td>
<td>$= 10$ units$^2$, $= 30$ units$^3$</td>
<td>$= 4\pi$ units$^2$, $\approx 75.4$ units$^3$</td>
</tr>
</tbody>
</table>

**Finding the Volume of Prisms and Cylinders**

Find the volume of the figure to the nearest tenth.

The base is a rectangle. Volume of a prism. Substitute for $B$ and $h$. Multiply.
Find the volume of each figure to the nearest tenth. Use 3.14 for \( \pi \).

**B**

\[ B = \pi (6^2) = 36\pi \text{ m}^2 \]

The base is a circle.

\[ V = Bh \]

Volume of a cylinder

\[ = 36\pi \cdot 15 \]

Substitute for \( B \) and \( h \).

\[ = 540\pi \approx 1695.6 \text{ m}^3 \]

Multiply.

**C**

\[ B = \frac{1}{2} \cdot 4 \cdot 7 = 14 \text{ ft}^2 \]

The base is a triangle.

\[ V = Bh \]

Volume of a prism

\[ = 14 \cdot 11 \]

Substitute for \( B \) and \( h \).

\[ = 154 \text{ ft}^3 \]

Multiply.

The formula for volume of a rectangular prism can be written as \( V = \ell wh \), where \( \ell \) is the length, \( w \) is the width, and \( h \) is the height.

### Example 2

**A**

A cereal box measures 6 in. by 2 in. by 9 in. Explain whether doubling only the length, width, or height of the box would double the amount of cereal the box holds.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Double Only the Length</th>
<th>Double Only the Width</th>
<th>Double Only the Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = \ell wh )</td>
<td>( V = (2\ell)wh )</td>
<td>( V = \ell (2w)h )</td>
<td>( V = \ell w(2h) )</td>
</tr>
<tr>
<td>( = 6 \cdot 2 \cdot 9 )</td>
<td>( = 12 \cdot 2 \cdot 9 )</td>
<td>( = 6 \cdot 4 \cdot 9 )</td>
<td>( = 6 \cdot 2 \cdot 18 )</td>
</tr>
<tr>
<td>( = 108 \text{ in}^3 )</td>
<td>( = 216 \text{ in}^3 )</td>
<td>( = 216 \text{ in}^3 )</td>
<td>( = 216 \text{ in}^3 )</td>
</tr>
</tbody>
</table>

The original box has a volume of 108 in\(^3\). You could double the volume to 216 in\(^3\) by doubling any one of the dimensions. So doubling only the length, width, or height would double the amount of cereal the box holds.

**B**

A can of corn has a radius of 2.5 in. and a height of 4 in. Explain whether doubling only the height of the can would have the same effect on the volume as doubling the radius.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Double Only the Height</th>
<th>Double Only the Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = \pi r^2h )</td>
<td>( V = \pi (2r)^2(2h) )</td>
<td>( V = \pi (2r)^2h )</td>
</tr>
<tr>
<td>( = 2.5^2 \pi \cdot 4 )</td>
<td>( = 2.5^2 \pi \cdot 8 )</td>
<td>( = 5^2 \pi \cdot 4 )</td>
</tr>
<tr>
<td>( = 25\pi \text{ in}^3 )</td>
<td>( = 50\pi \text{ in}^3 )</td>
<td>( = 100\pi \text{ in}^3 )</td>
</tr>
</tbody>
</table>

By doubling only the height, you would double the volume. By doubling only the radius, you would increase the volume four times the original.
**Music Application**

The Asano Taiko Company of Japan built the world's largest drum in 2000. The drum's diameter is 4.8 meters, and its height is 4.95 meters. Estimate the volume of the drum.

\[ d = 4.8 \approx 5, \quad h = 4.95 \approx 5 \quad \text{Use compatible numbers.} \]

\[ r = \frac{d}{2} = \frac{5}{2} = 2.5 \]

\[ V = (\pi r^2)h \quad \text{Volume of a cylinder} \]

\[ = (3.14)(2.5)^2 \cdot 5 \]

\[ = (3.14)(6.25)(5) \]

\[ = 19.625 \cdot 5 \]

\[ = 98.125 \approx 98 \]

The volume of the drum is approximately 98 m³.

To find the volume of a composite three-dimensional figure, find the volume of each part and add the volumes together.

**Example 4**

**Finding the Volume of Composite Figures**

Find the volume of the figure.

\[ V = Bh + Bh \]

\[ = (6)(9)(5) + \frac{1}{2}(6)(3)(9) \]

\[ = 270 + 81 \]

\[ = 351 \text{ cm}^3 \]

The volume of the figure is 351 cm³.

**Think and Discuss**

1. **Use models** to show that two rectangular prisms can have different heights but the same volume.

2. **Apply** your results from Example 2 to make a conjecture about changing dimensions in a triangular prism.

3. **Use a model** to describe what happens to the volume of a cylinder when the diameter of the base is tripled.
Exercises

10-2

**GUIDED PRACTICE**

Find the volume of each figure to the nearest tenth. Use 3.14 for π.

1. \( \text{6.3 cm} \times 21 \text{ cm} \times 7 \text{ cm} \)
2. \( \text{3 in.} \times 4 \text{ in.} \times 8 \text{ in.} \)
3. \( \frac{16 \text{ m}}{5 \text{ m}} \)

4. A can of juice has a radius 3 in. and a height 6 in. Explain whether tripling only the radius would triple the volume of the can.

5. Grain is stored in cylindrical structures called silos. Estimate the volume of a silo with diameter 11.1 feet and height 20 feet.

6. Find the volume of the barn at right.

**INDEPENDENT PRACTICE**

Find the volume of each figure to the nearest tenth. Use 3.14 for π.

7. \( \text{10 in.} \times 2 \text{ in.} \times 5 \text{ in.} \)
8. \( \text{1.5 cm} \times 11 \text{ cm} \times 13 \text{ cm} \)
9. \( \text{6 m} \times 13 \text{ m} \times 9 \text{ m} \)

10. A jewelry box measures 7 in. by 5 in. by 8 in. Explain whether increasing only the height 4 times, from 8 in. to 32 in., would increase the volume 4 times.

11. A toy box is 5.1 cm by 3.2 cm by 4.2 cm. Estimate the volume of the toy box.

12. Find the volume of the treehouse at right.

**PRACTICE AND PROBLEM SOLVING**

13. While Karim was at camp, his father sent him a care package. The box measured 10.2 in. by 19.9 in. by 4.2 in.
   a. Estimate the volume of the box.
   b. What might be the measurements of a box with twice its volume?
14. **Social Studies** The tablet held by the Statue of Liberty is approximately a rectangular prism with volume 1,107,096 in\(^3\). Estimate the thickness of the tablet.

15. **Life Science** The cylindrical Giant Ocean Tank at the New England Aquarium in Boston has a volume of 200,000 gallons.

   a. One gallon of water equals 231 cubic inches. How many cubic inches of water are in the Giant Ocean Tank? Write your answer in scientific notation.

   b. Use your answer from part a as the volume. The tank is 24 ft deep. Find the radius in feet of the Giant Ocean Tank.

16. **Reasoning** As many as 60,000 bees can live in 3 cubic feet of space. There are about 360,000 bees in a rectangular observation beehive that is 2 ft long by 3 ft high. What is the smallest possible width of the observation hive?

17. **What’s the Error?** A student read this statement in a book: “The volume of a triangular prism with height 15 in. and base area 20 in. is 300 in\(^3\).” Correct the error in the statement.

18. **Write About It** Explain why 1 cubic yard is the same as 27 cubic feet.

19. **Challenge** A 5-inch section of a hollow brick measures 12 inches tall and 8 inches wide on the outside. The brick is 1 inch thick. Find the volume of the brick, not the hollow interior.

---

### Spiral Standards Review

20. **Multiple Choice** Cylinder A has a radius of 6 cm and a height of 14 cm. Cylinder B has a radius that is half as long as cylinder A’s radius. If cylinder B has a height of 14 cm, what is the volume of cylinder B? Use 3.14 for \(\pi\).

   - **A** 393.5 cm\(^3\)  
   - **B** 395.6 cm\(^3\)  
   - **C** 422.3 cm\(^3\)  
   - **D** 791.3 cm\(^3\)

21. **Multiple Choice** A tractor trailer has dimensions of 13 feet by 53 feet by 8 feet. What is the volume of the trailer?

   - **A** 424 ft\(^3\)  
   - **B** 689 ft\(^3\)  
   - **C** 2756 ft\(^3\)  
   - **D** 5512 ft\(^3\)

---

Give the coordinates of each point after a reflection across the given axis.

**(Lesson 8-7)**

22. \((-3, 4); y\)-axis  
23. \((5, 9); x\)-axis  
24. \((6, -3); y\)-axis

25. Find the width of a rectangle with perimeter 14 inches and length 3 inches. What is the area of the rectangle? **(Lesson 9-1)**
The height of a pyramid or cone is measured from the vertex to the base along a line perpendicular to the base.

### Volume of Pyramids and Cones

**Why learn this?** You can use a formula to find the approximate volume of the Great Pyramid of Giza. (See Example 3.)

The volume of a pyramid or cone is given by the formula:

\[ V = \frac{1}{3}Bh \]

where \( B \) is the area of the base and \( h \) is the height.

### Rectangular pyramid

- Base: 4 cm x 9 cm
- Height: 12 cm

**Example 1**

**Finding the Volume of Pyramids and Cones**

Find the volume of the figure.

\[ B = \frac{1}{2} (4 \cdot 9) = 18 \text{ cm}^2 \]

\[ V = \frac{1}{3} \cdot 18 \cdot 9 = 54 \text{ cm}^3 \]
Find the volume of each figure. Use 3.14 for π.

\[ B = \pi(2^2) = 4\pi \text{ in}^2 \]
\[ V = \frac{1}{3} \cdot 4\pi \cdot 6 \]
\[ = 8\pi \]
\[ \approx 25 \text{ in}^3 \]

\[ B = 9 \cdot 7 = 63 \text{ ft}^2 \]
\[ V = \frac{1}{3} \cdot 63 \cdot 8 \]
\[ = 168 \text{ ft}^3 \]

\[ B = \pi(7^2) = 49\pi \text{ mm}^2 \]
\[ V = \frac{1}{3} \cdot 49\pi \cdot 8 \]
\[ = \frac{392}{3} \pi \]
\[ \approx 410.3 \text{ mm}^3 \]

**Social Studies Application**

The Great Pyramid of Giza is a square pyramid. Its height is 481 ft, and its base has side lengths of 756 ft. Find the volume of the pyramid.

**Step 1:** Find the area of the base.

\[ B = 756^2 \]
\[ = 571,536 \text{ ft}^2 \]

*The base is a square.*

*Multiply.*

**Step 2:** Find the volume.

\[ V = \frac{1}{3}Bh \]
\[ = \frac{1}{3} \cdot 571,536 \cdot 481 \]
\[ = 91,636,272 \text{ ft}^3 \]

*Write the formula.*

*Substitute for B and h.*

*Multiply.*

The volume of the pyramid is 91,636,272 ft\(^3\).

**Think and Discuss**

1. **Describe** two or more ways that you can change the dimensions of a rectangular pyramid to double its volume.

2. **Use a model** to compare the volume of a cube with 1 in. sides with a pyramid that is 1 in. high and has a 1 in. square base.
Find the volume of each figure to the nearest tenth. Use 3.14 for π.

1. 5 cm
2. 12 in.
3. 9.3 ft
4. 17 yd
5. 2.4 cm
6. 13
7. The Transamerica Pyramid in San Francisco has a base area of 22,000 ft² and a height of 853 ft. Find the volume of the building to the nearest thousand.

Find the volume of each figure to the nearest tenth. Use 3.14 for π.

8.
9. 5.5 m
10. 5 in.
11. 6.67 ft
12. 22 m
13. 13.5

A cone-shaped building has a diameter of 50 m and a height of 20 m. What is the volume of the building to the nearest hundredth?

Find the missing measure to the nearest tenth. Use 3.14 for π.

15. cone:
   radius = 4 in.
   height = __________ in
   volume = 100.5 in³
16. cylinder:
   radius = __________ m
   height = 2.5 m
   volume = 70.65 m³
17. triangular pyramid:
   base height = __________ ft
   base width = 8 ft
   height = 6 ft
   volume = 88 ft³
Find the volume of each figure to the nearest tenth. Use 3.14 for $\pi$.

18. [Diagram of cone with dimensions 5 in. base, 4 in. height]

19. [Diagram of pyramid with dimensions 10 cm base, 8 cm height, 6 cm depth]

20. **Architecture** The Pyramid of the Sun, in Teotihuacán, Mexico, is about 65 m tall and has a square base with side lengths of 225 m.
   a. What is the volume in cubic meters of the pyramid?
   b. How many cubic meters are in a cubic kilometer?
   c. What is the volume in cubic kilometers of the pyramid to the nearest thousandth?

21. **Estimation** Approximate the volume in cubic inches of an orange traffic cone with height 2 feet and diameter 10 inches by using 3 in place of $\pi$.

22. **What’s the Error?** A student says that the formula for the volume of a cylinder is the same as the formula for the volume of a pyramid, $\frac{1}{3}Bh$. What error did the student make?

23. **Architecture** The pyramid at the Louvre in Paris has a height of 72 ft and a square base with side lengths of 112 ft. What is the pyramid’s volume?

24. **Write About It** How would a cone’s volume be affected if you doubled the height? the radius? Use a model to help explain your answer.

25. **Challenge** The diameter of a cone is $x$ cm, the height is 18 cm, and the volume is $96\pi$ cm$^3$. Find the value of $x$.

---

26. **Multiple Choice** A pyramid has a rectangular base measuring 12 cm by 9 cm. Its height is 15 cm. What is the volume of the pyramid?
   - [A] 540 cm$^3$
   - [B] 405 cm$^3$
   - [C] 315 cm$^3$
   - [D] 270 cm$^3$

27. **Multiple Choice** A cone has diameter 12 cm and height 9 cm. Using 3.14 for $\pi$, find the volume of the cone to the nearest tenth.
   - [A] 1356.5 cm$^3$
   - [B] 339.1 cm$^3$
   - [C] 118.3 cm$^3$
   - [D] 56.5 cm$^3$

Solve. (Lesson 2-8)

28. $3x + 5 = 17$
29. $\frac{1}{2}x + 1\frac{1}{2} = 9$
30. $2.6 - x = 8.9$
31. $3.1 + 5.2x = -43.7$
32. Find the area of a circle with diameter 15 ft. (Lesson 9-4)
Quiz for Lessons 10-1 Through 10-3

10-1 Three-Dimensional Figures
Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

1. 2. 3.

10-2 Volume of Prisms and Cylinders
Find the volume of each figure to the nearest tenth. Use 3.14 for \( \pi \).

4. 5. 6.

7. A can is shaped like a cylinder. It is 5.2 cm wide and 2.3 cm tall. Find its volume to the nearest tenth. Use 3.14 for \( \pi \).

8. Find the volume of the composite figure at right.

10-3 Volume of Pyramids and Cones
Find the volume of each figure to the nearest tenth. Use 3.14 for \( \pi \).

9. 10. 11.

12. A cone has a radius of 2.5 cm and a height of 14 cm. What is the volume of the cone to the nearest hundredth? Use 3.14 for \( \pi \).
Focus on Problem Solving

Make a Plan

• Prioritize and sequence information

Some problems contain a lot of information. Read the entire problem carefully to be sure you understand all of the facts. You may need to read it over several times—perhaps aloud so that you can hear yourself say the words. Then decide which information is most important (prioritize). Is there any information that is absolutely necessary to solve the problem? This information is most important. Finally, put the information in order (sequence). Use comparison words like before, after, longer, shorter, and so on to help you. Write down the sequence before you try to solve the problem.

Read each problem below, and then answer the questions that follow.

1. Five friends are standing in line for the opening of a movie. They are in line according to their time of arrival. Tiffany arrived 3 minutes after Cedric. Roy took his place in line at 8:01 P.M. He was 1 minute behind Celeste and 7 minutes ahead of Tiffany. The first person arrived at 8:00 P.M. Blanca showed up 6 minutes after the first person. List the time of each person’s arrival.
   a. Whose arrival information helped you determine each arrival time?
   b. Can you determine the order without the time?
   c. List the friends’ order from the earliest to arrive to the last to arrive.

2. There are four children in the Putman family. Isabelle is half the age of Maxwell. Joe is 2 years older than Isabelle. Maxwell is 14. Hazel is twice Joe’s age and 4 years older than Maxwell. What are the ages of the children?
   a. Whose age must you figure out first before you can find Joe’s age?
   b. What are two ways to figure out Hazel’s age?
   c. List the Putman children from oldest to youngest.

California Standards

MR1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
Also covered: MG1.1
Nets of Prisms and Cylinders

Use with Lesson 10-4

A net is an arrangement of two-dimensional figures that can be folded to form a three-dimensional figure. You can explore the surface area of prisms and cylinders using nets.

**Activity 1**

1. Find four different-sized rectangular and triangular prisms. Follow these steps to make a net for each prism.
   a. Trace around each face of the prism on grid paper.
   b. Label the bases A and B. Continue labeling the lateral faces.
   c. Copy the tables shown. Fill in the information for each prism.

<table>
<thead>
<tr>
<th>Rectangular Prism</th>
<th>Triangular Prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face</td>
<td>Area</td>
</tr>
<tr>
<td>Base A</td>
<td></td>
</tr>
<tr>
<td>Base B</td>
<td></td>
</tr>
<tr>
<td>Lateral face C</td>
<td></td>
</tr>
<tr>
<td>Lateral face D</td>
<td></td>
</tr>
<tr>
<td>Lateral face E</td>
<td></td>
</tr>
<tr>
<td>Lateral face F</td>
<td></td>
</tr>
<tr>
<td><strong>Total Surface Area</strong></td>
<td><strong>Total Surface Area</strong></td>
</tr>
</tbody>
</table>

2. For each prism from 1, find the perimeter of a base. Then multiply the perimeter of the base by the prism's height. Finally, find the total area of the lateral faces.

**Think and Discuss**

1. In 2, how did the product of the base's perimeter and the prism's height compare with the sum of the areas of the lateral faces?
2. Write a rule for finding the surface area of any prism.

**Try This**

1. Use your rule from Think and Discuss 2 to find the surface area of two new prisms. Check your rule by following the steps in 1. Revise your rule as needed.
Activity 2

1. Find four different-sized cylinders. Follow these steps to make a net for each cylinder.
   a. Trace around the top of the cylinder on grid paper.
   b. Lay the cylinder on the grid paper so that it touches the circle, and mark its height. Then roll the cylinder one complete revolution, marking where the cylinder begins and ends. Draw a rectangle that has the same height as the cylinder and a width equal to one revolution of the cylinder.
   c. Trace the bottom of the cylinder so that it touches the bottom of the rectangle.
   d. Find the approximate area of each piece by counting squares.
   e. Add the areas to find the total surface area of the cylinder.
   f. Copy the table shown. Record the information in the table.

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Face</th>
<th>Area by Counting Squares</th>
<th>Area by Using Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Circular base A</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Circular base B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lateral face C</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Surface Area</td>
<td></td>
</tr>
</tbody>
</table>

2. Follow these steps for each cylinder from 1.
   a. Tape your pieces together to make a cylinder.
   b. Use area formulas to find the area of each base and the lateral face.
   c. Add the areas to find the total surface area of your net.
   d. Record the information in the table.

Think and Discuss

1. How did the area found by counting squares compare with the area found by using a formula?
2. How does the circumference of the base compare with the length of the lateral face?
3. Make a rule for finding the surface area of any cylinder.

Try This

1. Use your rule from Think and Discuss 3 to find the surface area of a new cylinder. Check your rule by following the steps in the activity. Revise your rule as needed.
10-4 Surface Area of Prisms and Cylinders

Why learn this? You can estimate the amount of reflective material you need to create an anamorphic image. (See Example 4.)

The surface area of a three-dimensional figure is the sum of the areas of all of its surfaces. You can use centimeter cubes to explore the surface area of prisms.

**Example 1: Finding Surface Area of Figures Built of Cubes**

Find the surface area of each figure. The figure is made up of congruent cubes.

**A**

![Diagram of a 3x3x3 cube]

Find the area of each view.

\[18 + 24 + 12 + 18 + 24 + 12 = 108\]

The surface area is 108 cm².

**B**

![Diagram of a 2x2x3 cube]

Find the area of each view.

\[8 + 10 + 8 + 8 + 10 + 8 = 52\]

The surface area is 52 cm².

The lateral faces of a prism are parallelograms that connect the bases. The lateral area of a prism is the sum of the areas of the lateral faces.
The surface area $S$ of a prism is twice the base area $B$ plus the lateral area $L$. The lateral area is the base perimeter $P$ times the height $h$.

**Finding Surface Area of Prisms**

Find the surface area of each prism to the nearest tenth.

The figure is a rectangular prism.

$$S = 2B + Ph$$

$$S = 2(3 \cdot 2) + (10)(5) = 62 \text{ units}^2$$

The figure is a triangular prism.

$$S = 2B + Ph$$

$$S = 2\left(\frac{1}{2} \cdot 9 \cdot 5.3\right) + (23)(7)$$

$$S = 208.7 \text{ cm}^2$$

The lateral surface of a cylinder is the curved surface that connects the bases.

The surface area $S$ of a cylinder is twice the base area $B$ plus the lateral area $L$. The lateral area is the base circumference $2\pi r$ times the height $h$.

**Surface Area of Prisms**

<table>
<thead>
<tr>
<th>Words</th>
<th>Formula</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The surface area $S$ of a prism is twice the base area $B$ plus the lateral area $L$. The lateral area is the base perimeter $P$ times the height $h$.</td>
<td>$S = 2B + L$ or $S = 2B + Ph$</td>
<td>![Image of a prism with dimensions]</td>
</tr>
<tr>
<td>$S = 2(3 \cdot 2) + (10)(5) = 62 \text{ units}^2$</td>
<td>![Image of a cylinder with dimensions]</td>
<td></td>
</tr>
</tbody>
</table>

**Surface Area of Cylinders**

<table>
<thead>
<tr>
<th>Words</th>
<th>Formula</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The surface area $S$ of a cylinder is twice the base area $B$ plus the lateral area $L$. The lateral area is the base circumference $2\pi r$ times the height $h$.</td>
<td>$S = 2\pi r^2 + 2\pi rh$</td>
<td>![Image of a cylinder with dimensions]</td>
</tr>
<tr>
<td>$S = 2\pi(5^2) + 2\pi(5)(6) \approx 345.4 \text{ units}^2$</td>
<td>![Image of a cylinder with dimensions]</td>
<td></td>
</tr>
</tbody>
</table>
Finding Surface Area of Cylinders

Find the surface area of the cylinder to the nearest tenth. Use 3.14 for \( \pi \).

\[
S = 2\pi r^2 + 2\pi rh
\]

Surface area of a cylinder

\[
= 2\pi (2^2) + 2\pi (2)(8)
\]

Substitute for \( r \) and \( h \).

\[
= 40\pi m^2
\]

\[
= 125.6 m^2
\]

Art Application

An anamorphic image is a distorted picture that becomes recognizable when reflected onto a cylindrical mirror. A cylinder is 49 mm in diameter and 107 mm tall. Estimate the amount of reflective material you would need to cover the cylinder.

The cylinder’s diameter is about 50 mm, and its height is about 100 mm.

\[
L = 2\pi rh
\]

Only the lateral surface needs to be covered.

\[
= 2\pi (25)(100)
\]

diameter \( \approx 50 \text{ mm} \), so \( r \approx 25 \text{ mm} \)

\[
= 15,700 \text{ mm}^2
\]

Think and Discuss

1. Explain how finding the surface area of a cylindrical drinking glass would be different from finding the surface area of a cylinder.

10-4 Exercises

GUIDED PRACTICE

Find the surface area of each figure. The figure is made up of congruent cubes.

1. 2. 3.
Find the surface area of each prism to the nearest tenth.

4. \( \text{Base: } 14 \text{ cm} \times 8 \text{ cm} \), \( \text{Height: } 3 \text{ cm} \)

5. \( \text{Base: } 3 \text{ m} \times 3 \text{ m} \), \( \text{Height: } 2.6 \text{ m} \)

6. \( \text{Base: } 5 \text{ in.} \times 7 \text{ in.} \), \( \text{Height: } 5 \text{ in.} \)

Find the surface area of each cylinder to the nearest tenth. Use 3.14 for \( \pi \).

7. \( \text{Radius: } 4 \text{ in.} \), \( \text{Height: } 12 \text{ in.} \)

8. \( \text{Radius: } 10 \text{ m} \), \( \text{Height: } 18 \text{ m} \)

9. \( \text{Radius: } 6 \text{ cm} \), \( \text{Height: } 15 \text{ cm} \)

Tilly is covering the lateral surface area of a can with colored paper. The can is 8 in. tall and has a radius of 2 in. Estimate the amount of paper she needs.

INDEPENDENT PRACTICE

Find the surface area of each figure. The figure is made up of congruent cubes.

11. \( \text{Volume: } 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} \)

12. \( \text{Volume: } 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} \)

13. \( \text{Volume: } 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} \)

Find the surface area of each prism to the nearest tenth.

14. \( \text{Base: } 4 \text{ m} \times 4 \text{ m} \), \( \text{Height: } 5 \text{ m} \)

15. \( \text{Base: } 15 \text{ mm} \times 8 \text{ mm} \), \( \text{Height: } 26 \text{ mm} \)

16. \( \text{Base: } 5 \text{ ft} \times 1 \text{ ft} \), \( \text{Height: } 3 \text{ ft} \)

Find the surface area of each cylinder to the nearest tenth. Use 3.14 for \( \pi \).

17. \( \text{Radius: } 6 \text{ cm} \), \( \text{Height: } 15 \text{ cm} \)

18. \( \text{Radius: } 4 \text{ mm} \), \( \text{Height: } 3 \text{ mm} \)

19. \( \text{Radius: } 4 \text{ mm} \), \( \text{Height: } 10 \text{ yd} \)

Frank is wrapping a present. The box measures 6.2 cm by 9.9 cm by 5.1 cm. Estimate the amount of wrapping paper, not counting overlap, that Frank needs.
PRACTICE AND PROBLEM SOLVING

Find the surface area of each figure to the nearest tenth. Use 3.14 for \( \pi \).

21. cylinder: \( d = 30 \text{ mm}, h = 49 \text{ mm} \)
22. rectangular prism: \( 5 \frac{1}{4} \text{ in. by 8 in. by 12 in.} \)

Find the missing dimension in each figure with the given surface area.

23. \( S = 256 \text{ m}^2 \) 12 m
24. \( S = 120 \pi \text{ cm}^2 \) 5 cm

25. Multi-Step Jesse makes 12 in. by 6 in. by 8 in. rectangular glass aquariums. Glass costs $0.08 per square inch. How much will glass for one aquarium cost?

26. Reasoning A cylinder has diameter 10 in. and height 4 in. Explain whether doubling only the height would have the same effect on the surface area as doubling only the radius. What happens if you double both dimensions?

27. Choose a Strategy Which of the following nets can be folded into the given three-dimensional figure?

28. Write About It Explain how you would find the side lengths of a cube with a surface area of 512 \( \text{ft}^2 \).

29. Challenge The rectangular wood block shown has a hole with diameter 4 cm drilled through its center. What is the total surface area of the block?

Spiral Standards Review

30. Multiple Choice Find the surface area of a cylinder with radius 5 feet and height 3 feet. Use 3.14 for \( \pi \).

\[ A \rightarrow 125.6 \text{ ft}^2 \quad B \rightarrow 150.72 \text{ ft}^2 \quad C \rightarrow 172.7 \text{ ft}^2 \quad D \rightarrow 251.2 \text{ ft}^2 \]

31. Gridded Response A rectangular prism has dimensions 2 meters by 4 meters by 18 meters. Find the surface area, in square meters, of the prism.

Add or subtract. (Lesson 2-3)

32. \(-0.4 + 0.7\) 33. \(1.35 - 5.6\) 34. \(-0.01 - 0.25\) 35. \(-0.65 + (-1.12)\)

Find the area of each figure with the given dimensions. (Lesson 9-2)

36. triangle: \( b = 3 \frac{1}{2}, h = 5 \)
37. triangle: \( b = 17, h = 13 \)
38. trapezoid: \( b_1 = 3.4, b_2 = 6.6, h = 1.8 \)
You can estimate the surface area of cones using models and nets.

**Activity**

1. Turn a cone-shaped paper cup upside down and trace the bottom onto grid paper. This represents the base of a cone.

2. Use scissors to cut a straight line from the edge of the cup to the vertex. Then flatten the shape made by the cut cup.

3. Trace the flattened cup onto the grid paper so that the curved edge touches the edge of the circle you drew for the base. This is a net for a cone.

4. Estimate the surface area of the cone by adding the number of whole and almost-whole squares covered by the shapes and half the number of half-squares covered by the shapes.

   Number of whole and almost-whole squares: 68
   Number of half-squares: 10

   \[68 + \frac{1}{2}(10) = 73\]

   The surface area of the cone is about 73 square units.

**Think and Discuss**

1. The flattened shape made by cutting the cup is a portion of another shape. What shape do you think that is? Explain.

**Try This**

1. Find another cone-shaped object, such as a party hat, and repeat the steps in the Activity. Estimate the surface area of a cone the size of the object.
The slant height of a pyramid or cone is measured along its lateral surface.

The base of a regular pyramid is a regular polygon, and the lateral faces are all congruent.

In a right cone, a line perpendicular to the base through the vertex passes through the center of the base.

### SURFACE AREA OF PYRAMIDS AND CONES

<table>
<thead>
<tr>
<th><strong>Words</strong></th>
<th><strong>Formula</strong></th>
<th><strong>Numbers</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pyramid:</strong> The surface area ( S ) of a regular pyramid is the base area ( B ) plus the lateral area ( L ). The lateral area is one-half the base perimeter ( P ) times the slant height ( \ell ).</td>
<td>( S = B + L ) or ( S = B + \frac{1}{2}P\ell )</td>
<td>( S = (12 \cdot 12) + \frac{1}{2}(48)(8) = 336 \text{ units}^2 )</td>
</tr>
<tr>
<td><strong>Cone:</strong> The surface area ( S ) of a right cone is the base area ( B ) plus the lateral area ( L ). The lateral area is one-half the base circumference ( 2\pi r ) times the slant height ( \ell ).</td>
<td>( S = B + L ) or ( S = \pi r^2 + \pi r\ell )</td>
<td>( S = \pi(2^2) + \pi(2)(5) = 14\pi \approx 43.98 \text{ units}^2 )</td>
</tr>
</tbody>
</table>

### E X A M P L E 1 Finding Surface Area

Find the surface area of the figure to the nearest tenth.

\[ S = B + \frac{1}{2}P\ell \]
\[ = (2.5 \cdot 2.5) + \frac{1}{2}(10)(3) \]
\[ = 21.25 \text{ in}^2 \]
Find the surface area of the figure to the nearest tenth. Use 3.14 for $\pi$.

$$S = \pi r^2 + \pi r \ell$$

$$= \pi (4)^2 + \pi (4)(7)$$

$$= 16\pi + 28\pi$$

$$= 44\pi \approx 138.2 \text{ m}^2$$

**Exploring the Effects of Changing Dimensions**

A cone has diameter 6 in. and slant height 4 in. Explain whether doubling only the slant height would have the same effect on the surface area as doubling only the radius. Use 3.14 for $\pi$.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Double the Slant Height</th>
<th>Double the Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = \pi r^2 + \pi r \ell$</td>
<td>$S = \pi r^2 + \pi r (2\ell)$</td>
<td>$S = \pi (2r)^2 + \pi (2r) \ell$</td>
</tr>
<tr>
<td>$= \pi (3)^2 + \pi (3)(4)$</td>
<td>$= \pi (3)^2 + \pi (3)(8)$</td>
<td>$= \pi (6)^2 + \pi (6)(4)$</td>
</tr>
<tr>
<td>$= 21\pi \text{ in}^2 \approx 66.0 \text{ in}^2$</td>
<td>$= 33\pi \text{ in}^2 \approx 103.6 \text{ in}^2$</td>
<td>$= 60\pi \text{ in}^2 \approx 188.4 \text{ in}^2$</td>
</tr>
</tbody>
</table>

They would not have the same effect. Doubling the radius would increase the surface area more than doubling the slant height.

**Life Science Application**

An ant lion pit is an inverted cone with the dimensions shown. What is the lateral surface area of the pit?

The slant height, radius, and depth of the pit form a right triangle.

$$a^2 + b^2 = \ell^2$$

$$2.5^2 + 2^2 = \ell^2$$

$$10.25 = \ell^2$$

$$\ell = 3.2$$

Lateral surface area

$$L = \pi r \ell$$

$$= \pi (2.5)(3.2) \approx 25.1 \text{ cm}^2$$

**Think and Discuss**

1. Compare the formula for surface area of a pyramid to the formula for surface area of a cone.

2. Explain how you would find the slant height of a square pyramid with base edge length 6 cm and height 4 cm.
1. Find the surface area of each figure to the nearest tenth. Use 3.14 for $\pi$.

See Example 1

1. 2.

5 m 8 m 5 m

3. 4.5 in. 3 in. 3 in.

1.5 ft 5 ft

4. A cone has diameter 12 in. and slant height 9 in. Explain whether doubling both dimensions would double the surface area.

5. The rooms at the Wigwam Village Motel in Cave City, Kentucky, are cones about 20 ft high and have a diameter of about 20 ft. Estimate the lateral surface area of a room.

6. Find the surface area of each figure to the nearest tenth. Use 3.14 for $\pi$.

See Example 2

6. 7.

5.5 in. 4 in. 4 in.

4 in. 4 in.

7. 6 mm 4 mm

8.

5. A regular square pyramid has a base with 12 yd sides and slant height 5 yd. Explain whether doubling both dimensions would double the surface area.

9. In the late 1400s, Leonardo da Vinci designed a parachute shaped like a pyramid. His design called for a tent-like structure made of linen, measuring 21 feet on each side and 12 feet high. Estimate how much material would be needed to make the parachute.

10. Find the surface area of each figure with the given dimensions. Use 3.14 for $\pi$.

See Example 3

1. 4.5 in. 3 in. 3 in.

11. regular triangular pyramid:

base area = 0.06 km$^2$

base perimeter = 0.8 km

12. cone:

radius = 5 mi

slant height = 0.3 km

slant height = 13 mi
13. **Science** When the Moon is between the Sun and Earth, it casts a conical shadow called the *umbra*. If the shadow is 2140 mi in diameter and 260,955 mi along the edge, what is the lateral area of the umbra? Give your answer in terms of $\pi$.

14. **Social Studies** The Pyramid Arena in Memphis, Tennessee, is 321 feet tall and has a square base with side length 200 yards. What is the lateral area of the pyramid in feet?

15. The table shows the dimensions of three square pyramids.
   
   a. Complete the table.
   
   b. Which pyramid has the least lateral area? What is its lateral area?
   
   c. Which pyramid has the greatest volume? What is its volume?

<table>
<thead>
<tr>
<th>Pyramid</th>
<th>Height</th>
<th>Slant Height</th>
<th>Side of Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khufu</td>
<td>612</td>
<td>756</td>
<td>756</td>
</tr>
<tr>
<td>Khafre</td>
<td>471</td>
<td>588</td>
<td>704</td>
</tr>
<tr>
<td>Menkaure</td>
<td>216</td>
<td>346</td>
<td>346</td>
</tr>
</tbody>
</table>

16. **Write a Problem** A cone has a diameter of 4 in. and a slant height of 11 in. Write and solve a problem about the cone.

17. **Write About It** The height and base dimensions of a cone are known. Explain how to find the slant height.

18. **Challenge** The oldest pyramid is said to be the Step Pyramid of King Zoser, built around 2650 B.C. in Saqqara, Egypt. The base is a rectangle that measures 358 ft by 411 ft, and the height of the pyramid is 204 ft. Find the lateral area of the pyramid.

19. **Multiple Choice** Find the surface area of a triangular pyramid with base area 12 square meters, base perimeter 24 meters, and slant height 8 meters.
   
   A. $72 \text{ m}^2$  
   B. $108 \text{ m}^2$  
   C. $204 \text{ m}^2$  
   D. $2304 \text{ m}^2$

20. **Gridded Response** What is the lateral surface area of a cone with diameter 12 centimeters and slant height 6 centimeters? Use 3.14 for $\pi$.

21. $-4(6 + 8)$  
22. $3(-5 - 4)$  
23. $-2(4) - 9$  
24. $-6(8 - 9)$

Find the volume of each rectangular prism. **(Lesson 10-2)**

25. length 5 ft, width 3 ft, height 8 ft  
26. length 2.5 m, width 3.5 m, height 7 m
A sphere is the set of points in three dimensions that are a fixed distance from a given point, the center. Earth is not a perfect sphere, but it has been molded by gravitational forces into an approximately spherical shape.

A plane that intersects a sphere through its center divides the sphere into two halves, or hemispheres. The edge of a hemisphere is a great circle.

The volume of a hemisphere is exactly halfway between the volume of a cone and the volume of a cylinder with the same radius \( r \) and height equal to \( r \).

The volume \( V \) of a sphere is \( \frac{4}{3} \pi r^3 \) times the cube of the radius \( r \).

**VOLUME OF A SPHERE**

<table>
<thead>
<tr>
<th>Words</th>
<th>Formula</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The volume ( V ) of a sphere is ( \frac{4}{3} \pi r^3 ) times the cube of the radius ( r ).</td>
<td>( V = \frac{4}{3} \pi r^3 )</td>
<td>( V = \frac{4}{3} \pi (3^3) = \frac{108}{3} \pi = 36 \pi \approx 113.1 \text{ units}^3 )</td>
</tr>
</tbody>
</table>

**EXAMPLE 1**

Finding the Volume of a Sphere

Find the volume of a sphere with radius 9 ft, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).

\[
V = \frac{4}{3} \pi r^3
\]

\[
= \frac{4}{3} \pi (9)^3
\]

\[
= 972\pi \text{ ft}^3 \approx 3052.1 \text{ ft}^3
\]
The surface area of a sphere is four times the area of a great circle.

### Finding Surface Area of a Sphere

Find the surface area, both in terms of $\pi$ and to the nearest tenth. Use 3.14 for $\pi$.

**Surface area of a sphere**

$$S = 4\pi r^2$$

**EXAMPLE**

Find the surface area of a sphere with radius 4 mm.

$$S = 4\pi (4^2)$$

$$= 64\pi \text{ mm}^2 \approx 201.0 \text{ mm}^2$$

### Comparing Volumes and Surface Areas

Compare the volume and surface area of a sphere with radius 42 cm with that of a rectangular prism measuring 56 $\times$ 63 $\times$ 88 cm.

**Sphere:**

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (42)^3$$

$$\approx \left(\frac{4}{3}\right)\left(\frac{22}{7}\right)(74,088)$$

$$\approx 310,464 \text{ cm}^3$$

**Rectangular prism:**

$$V = \ell \omega h$$

$$= (56)(63)(88)$$

$$= 310,464 \text{ cm}^3$$

**Surface area of a sphere**

$$S = 4\pi r^2 = 4\pi (42)^2$$

$$= 7056\pi$$

$$\approx 7056\left(\frac{22}{7}\right) \approx 22,176 \text{ cm}^2$$

**Surface area of a rectangular prism**

$$S = 2\ell \omega + 2\ell h + 2\omega h$$

$$= 2(56)(63) + 2(56)(88) + 2(63)(88)$$

$$= 28,000 \text{ cm}^2$$

The sphere and the prism have approximately the same volume, but the prism has a larger surface area.

### Think and Discuss

1. **Compare** the area of a great circle with the surface area of a sphere.

2. **Explain** which would hold the most water: a bowl in the shape of a hemisphere with radius $r$, a cylindrical glass with radius $r$ and height $r$, or a conical drinking cup with radius $r$ and height $r.$
10-6 Exercises

GUIDED PRACTICE

See Example 1
Find the volume of each sphere, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).
1. \( r = 3 \text{ cm} \)  
2. \( r = 12 \text{ ft} \)  
3. \( d = 3.4 \text{ m} \)  
4. \( d = 10 \text{ mi} \)

See Example 2
Find the surface area of each sphere, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).
5. \( \text{diameter} = 1 \text{ in.} \)  
6. \( \text{diameter} = 7.7 \text{ mm} \)  
7. \( \text{diameter} = 8 \text{ cm} \)  
8. \( \text{diameter} = 17 \text{ yd} \)

See Example 3
9. Compare the volume and surface area of a sphere with radius 4 in. with that of a cube with sides measuring 6.45 in.

INDEPENDENT PRACTICE

See Example 1
Find the volume of each sphere, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).
10. \( r = 14 \text{ ft} \)  
11. \( r = 5.7 \text{ cm} \)  
12. \( d = 26 \text{ mm} \)  
13. \( d = 2 \text{ in.} \)

See Example 2
Find the surface area of each sphere, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).
14. \( \text{diameter} = 4 \text{ ft} \)  
15. \( \text{diameter} = 7.2 \text{ m} \)  
16. \( \text{diameter} = 7 \text{ km} \)  
17. \( \text{diameter} = 20 \text{ cm} \)

See Example 3
18. Compare the volume and surface area of a sphere with diameter 5 ft with that of a cylinder with height 2 ft and a base with radius 3 ft.

PRACTICE AND PROBLEM SOLVING

Extra Practice

Find the missing measurements of each sphere, both in terms of \( \pi \) and to the nearest hundredth. Use 3.14 for \( \pi \).
19. radius = 6.5 in.  
   volume =  
   surface area = 169\pi \text{ in}^2

20. radius = 11.2 m  
    volume = 1873.24\pi \text{ m}^3  
    surface area =

21. diameter = 6.8 yd  
    volume =  
    surface area =

22. radius =  
    diameter = 22 in.  
    surface area =

23. Reasoning Use models of a sphere, a cylinder, and two cones. The sphere and cylinder have the same diameter and height. The cones have the same diameter and half the height of the sphere. Describe the relationship between the volumes of these shapes.
Eggs come in many different shapes. The eggs of birds that live on cliffs are often extremely pointed to keep the eggs from rolling. Other birds, such as great horned owls, have eggs that are nearly spherical. Turtles and crocodiles also have nearly spherical eggs, and the eggs of many dinosaurs were spherical.

24. To lay their eggs, green turtles travel hundreds of miles to the beach where they were born. The eggs are buried on the beach in a hole about 40 cm deep. The eggs are approximately spherical, with an average diameter of 4.5 cm, and each turtle lays an average of 113 eggs at a time. Estimate the total volume of eggs laid by a green turtle at one time.

25. Fossilized embryos of dinosaurs called titanosaurid sauropods have recently been found in spherical eggs in Patagonia. The eggs were 15 cm in diameter, and the adult dinosaurs were more than 12 m in length. Find the volume of an egg.

26. Hummingbirds lay eggs that are nearly spherical and about 1 cm in diameter. Find the surface area of an egg.

27. **Challenge** A spherical-shaped ostrich egg has about the same volume as a sphere with a diameter of 5 inches. If the shell is about \( \frac{1}{12} \) inch thick, estimate the volume of just the shell, not including the interior of the egg.

---

**Spiral Standards Review**

**NS2.4, MG2.1**

28. **Multiple Choice** The surface area of a sphere is 50.24 square centimeters. Find the length of the diameter. Use 3.14 for \( \pi \).

- A 1 cm  
- B 2 cm  
- C 2.5 cm  
- D 4 cm

29. **Gridded Response** Find the surface area, in square feet, of a sphere with radius 3 feet. Use 3.14 for \( \pi \).

Simplify. (Lesson 4-6)

- 30. \( \sqrt{144} \)  
- 31. \( \sqrt{64} \)  
- 32. \( \sqrt{169} \)  
- 33. \( \sqrt{225} \)  
- 34. \( \sqrt{1} \)

Find the surface area of each figure. Use 3.14 for \( \pi \). (Lesson 10-5)

- 35. a square pyramid with base 13 m by 13 m and slant height 7.5 m
- 36. a cone with a diameter 90 cm and slant height 125 cm
A packaging company offers a supply of cube boxes with measurements shown. What is the volume and surface area of each of these boxes?

<table>
<thead>
<tr>
<th>Edge Length</th>
<th>1 ft</th>
<th>2 ft</th>
<th>3 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>$1 \times 1 \times 1 = 1$ ft$^3$</td>
<td>$2 \times 2 \times 2 = 8$ ft$^3$</td>
<td>$3 \times 3 \times 3 = 27$ ft$^3$</td>
</tr>
<tr>
<td>Surface Area</td>
<td>$6 \times 1 \times 1 = 6$ ft$^2$</td>
<td>$6 \times 2 \times 2 = 24$ ft$^2$</td>
<td>$6 \times 3 \times 3 = 54$ ft$^2$</td>
</tr>
</tbody>
</table>

Corresponding edge lengths of any two cubes are in proportion to each other because the cubes are similar. However, volumes and surface areas do not have the same scale factor as edge lengths.

Each edge of the 2 ft cube is 2 times as long as each edge of the 1 ft cube. However, the cube's volume, or capacity, is $2^3 = 8$ times as large, and its surface area is $2^2 = 4$ times as large as the 1 ft cube's.

**Example 1**

Scaling Models That Are Cubes

A 6 cm cube is built from small cubes, each 2 cm on an edge. Compare the following values.

A. the edge lengths of the large and small cubes

\[
\frac{6 \text{ cm cube}}{2 \text{ cm cube}} \rightarrow \frac{6 \text{ cm}}{2 \text{ cm}} = 3 \quad \text{Ratio of corresponding edges}
\]

The edge length of the large cube is 3 times that of the small cube.

B. the surface areas of the two cubes

\[
\frac{6 \text{ cm cube}}{2 \text{ cm cube}} \rightarrow \frac{216 \text{ cm}^2}{24 \text{ cm}^2} = 9 \quad \text{Ratio of corresponding areas}
\]

The surface area of the large cube is $3^2 = 9$ times that of the small cube.

C. the volumes of the two cubes

\[
\frac{6 \text{ cm cube}}{2 \text{ cm cube}} \rightarrow \frac{216 \text{ cm}^3}{8 \text{ cm}^3} = 27 \quad \text{Ratio of corresponding volumes}
\]

The volume of the large cube is $3^3 = 27$ times that of the small cube.
The previous example suggests that the following ratios are true for similar three-dimensional figures.

### Ratios of Similar Solids
- If two three-dimensional figures are similar by a scale of $k$, then the surface areas of the figures have a ratio of $k^2$.
- If two three-dimensional figures are similar by a scale of $k$, then the volumes of the figures have a ratio of $k^3$.

### Example 2
**Finding Surface Area and Volume of Similar Solids**

#### A
The surface area of a box is 27 in$^2$. What is the surface area of a similar box that is larger by a scale factor of 5?

\[ S = 27 \cdot 5^2 \quad \text{Multiply by the square of the scale factor.} \]

\[ = 27 \cdot 25 \quad \text{Simplify the power.} \]

\[ = 675 \text{ in}^2 \quad \text{Multiply.} \]

#### B
The volume of a bucket is 6237 in$^3$. What is the volume of a similar bucket that is smaller by a scale factor of $\frac{1}{3}$?

\[ V = 6237 \cdot \left(\frac{1}{3}\right)^3 \quad \text{Multiply by the cube of the scale factor.} \]

\[ = 6237 \cdot \frac{1}{27} \quad \text{Simplify the power.} \]

\[ = 231 \text{ in}^3 \quad \text{Multiply.} \]

### Example 3
**Business Application**

A machine fills a cube box that has edge lengths of 1 ft with shampoo samples in 3 seconds. How long does it take the machine to fill at the same rate a cube box that has edge lengths of 4 ft?

\[ V = 4 \text{ ft} \cdot 4 \text{ ft} \cdot 4 \text{ ft} = 64 \text{ ft}^3 \quad \text{Find the volume of the larger box.} \]

\[ \frac{3}{1 \text{ ft}^3} = \frac{x}{64 \text{ ft}^3} \quad \text{Set up a proportion and solve.} \]

\[ 3 \cdot 64 = x \quad \text{Cross multiply.} \]

\[ 192 = x \quad \text{Calculate the fill time.} \]

It takes 192 seconds to fill the larger box.

### Think and Discuss

1. **Describe** how the volume of a model compares to the original object if the scale factor of the model is 1:2.

2. **Explain** one possible way to double the surface area of a rectangular prism.
**GUIDED PRACTICE**

1. **See Example 1**
   An 8 in. cube is built from small cubes, each 2 in. on an edge.
   Compare the following values.
   1. the edge lengths of the large and small cubes
   2. the surface areas of the two cubes
   3. the volumes of the two cubes

2. **See Example 2**
   4. The surface area of a box is 10.4 cm². What is the surface area of a similar box that is larger by a scale factor of 3?
   5. The volume of a cylinder is about 523 cm³. What is the volume, to the nearest tenth, of a similar cylinder that is smaller by a scale factor of \( \frac{1}{4} \)?

3. **See Example 3**
   6. A 3 ft by 1 ft by 1 ft fish tank in the shape of a rectangular prism drains in 3 min. How long would it take a 7 ft by 4 ft by 4 ft fish tank to drain at the same rate?

**INDEPENDENT PRACTICE**

1. **See Example 1**
   A 6 m cube is built from small cubes, each 3 m on an edge. Compare the following values.
   7. the edge lengths of the large and small cubes
   8. the surface areas of the two cubes
   9. the volumes of the two cubes

2. **See Example 2**
   10. The surface area of a car frame is about 200 ft². What is the surface area, to the nearest tenth, of a model of the car that is smaller by a scale factor of \( \frac{1}{2} \)?
   11. The volume of an ice chest is 2160 in³. What is the volume of a similar ice chest that is larger by a scale factor of 2.5?

3. **See Example 3**
   12. An aboveground pool 5 ft tall with a diameter of 40 ft is filled with water in 50 minutes. How long will it take to fill an aboveground pool that is 6 ft tall with a diameter of 36 ft?

**PRACTICE AND PROBLEM SOLVING**

**Extra Practice**


For each cube, a reduced scale model is built using a scale factor of \( \frac{1}{2} \). Find the length of the model and the number of 1 cm cubes used to build it.

13. a 2 cm cube
14. a 6 cm cube
15. an 18 cm cube
16. a 4 cm cube
17. a 14 cm cube
18. a 16 cm cube
19. What is the volume in cubic centimeters of a 1 m cube?
20. **Art** A sand castle requires 3 pounds of sand. How much sand would be required to double all the dimensions of the sand castle?
21. A kitchen sink measures 21 in. by 16 in. by 8 in. It takes 4 minutes 30 seconds to fill with water. A smaller kitchen sink takes 4 min 12 seconds to fill with water.

a. What is the volume of the smaller kitchen sink?

b. About how many gallons of water does the smaller kitchen sink hold? (Hint: 1 gal = 231 in³)

22. Recreation If it took 100,000 Lego® blocks to build a cylindrical monument with a 5 m diameter, about how many Legos would be needed to build a monument with an 8 m diameter and the same height?

23. Reasoning If you double the length of each edge of a cube, are the surface area and volume of the cube also doubled? Explain.

24. Choose a Strategy Six 1 cm cubes are used to build a solid. How many cubes are used to build a scale model of the solid with a scale factor of 2 to 1? Describe the tools and techniques you used.

A 12 cubes  B 24 cubes  C 48 cubes  D 144 cubes

25. Write About It If the scale factor of a model is 1:5, what is the relationship between the volume of the original object and the volume of the model?

26. Challenge To double the volume of a rectangular prism, what number is multiplied by each of the prism’s linear dimensions? Give your answer to the nearest hundredth.

27. Multiple Choice A 9-inch cube is built from small cubes, each 1 inch on an edge. What is the ratio of the volume of the larger cube to the volume of the smaller cube?

A 1:9  B 9:1  C 81:1  D 729:1

28. Extended Response A 5-inch cube is built from small cubes, each 1 inch on an edge. Compare the edge lengths, surface areas, and volumes of the large and the small cubes.

Solve. (Lesson 1-9)

29. 3 + 4x = 35
30. −y − 6 = 8
31. 21 = 5w + 11
32. −24 = 10b − 4

Find the surface area of each sphere to the nearest tenth. Use 3.14 for π. (Lesson 10-6)

33. radius 5 mm
34. radius 12.2 ft
35. diameter 4 in.
36. diameter 20 cm
Quiz for Lessons 10-4 Through 10-7

10-4 Surface Area of Prisms and Cylinders
Find the surface area of each figure to the nearest tenth. Use 3.14 for $\pi$.
1. 2. 3.

10-5 Surface Area of Pyramids and Cones
Find the surface area of each figure to the nearest tenth. Use 3.14 for $\pi$.
4. 5. 6.

10-6 Spheres
Find the volume and surface area of each sphere to the nearest tenth. Use 3.14 for $\pi$.
7. 8. 9.

10-7 Scaling Three-Dimensional Figures
10. The surface area of a cylinder is 109 cm$^2$. What is the surface area of a similar cylinder that is smaller by a scale factor of $\frac{1}{3}$?

11. The volume of a cube is 35 ft$^3$. What is the volume of a similar cube that is larger by a scale factor of 9?
It’s a Wrap!  Kim and Miguel are raising money for their school track team by running a gift-wrapping service at the mall. Customers can also have their gifts boxed for shipping. Kim and Miguel have rolls of gift wrap, shipping boxes, cardboard, and packing peanuts.

1. A customer wants to wrap and ship a gift that is in the shape of a rectangular prism. The dimensions of the gift are 10 in. by 15 in. by 4 in. How many square inches of wrapping paper are needed to wrap the gift?

2. Kim chooses a shipping box that is 18 in. by 12 in. by 6 in. After the gift is placed inside the box, she will fill the empty space with packing peanuts. How many cubic inches of packing peanuts will Kim need? Explain.

3. Another customer wants to ship a large cone-shaped art piece made out of recycled glass. The figure shows the dimensions of the conic art. Miguel decides to use poster board to make a cylindrical container that is just large enough to hold the art. How much poster board will he need?

4. Once the conic art is placed in the cylindrical container, how many cubic inches of packing peanuts will be needed to fill the empty space?
Planes in Space

Some three-dimensional figures can be generated by plane figures.

Experiment with a circle first. Move the circle around. See if you recognize any three-dimensional shapes.

If you rotate a circle around a diameter, you get a sphere.

If you translate a circle up along a line perpendicular to the plane that the circle is in, you get a cylinder.

If you rotate a circle around a line outside the circle but in the same plane as the circle, you get a donut shape called a torus.

Draw or describe the three-dimensional figure generated by each plane figure.

1. a square translated along a line perpendicular to the plane it is in
2. a rectangle rotated around one of its edges
3. a right triangle rotated around one of its legs

Magic Cubes

Four magic cubes are used in this fun puzzle. A complete set of rules and nets for making the cubes can be found online. Each side of the four cubes has the number 1, 2, 3, or 4 written on it. The object of the game is to stack the cubes so that the numbers along each side of the stack add up to 10. No number can be repeated along any side of the stack.
PROJECT  The Tube Journal

Use this journal to take notes on perimeter, area, and volume. Then roll up the journal and store it in a tube for safekeeping!

Directions

1. Start with several sheets of paper that measure 8½ inches by 11 inches. Cut an inch off the end of each sheet so they measure 8½ inches by 10 inches.

2. Stack the sheets and fold them in half lengthwise to form a journal that is approximately 4¼ inches by 10 inches. Cover the outside of the journal with decorative paper, trim it as needed, and staple everything together along the edge. Figure A

3. Punch a hole through the journal in the top left corner. Tie a 6-inch piece of twine or yarn through the hole. Figure B

4. Use glue to cover a cardboard tube with decorative paper. Then write the name and number of the chapter on the tube.

Taking Note of the Math

Use your journal to take notes on perimeter, area, and volume. Then roll up the journal and store it in the cardboard tube. Be sure the twine hangs out of the tube so that the journal can be pulled out easily.
Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?____ has two parallel, congruent circular bases connected by a curved surface.

2. The sum of the areas of the surfaces of a three-dimensional figure is called the ___?____.

3. A(n) ___?____ has one circular base and a curved surface.

---

### 10-1 Three-Dimensional Figures  (pp. 480–483)

**Example**

Name the figure.

There are two bases that are hexagons.

The figure is a hexagonal prism.

**Exercises**

Name each figure.

4.  

5.  

---

### 10-2 Volume of Prisms and Cylinders  (pp. 485–489)

**Example**

Find the volume of the prism.

\[ V = Bh \]

\[ V = (15 \cdot 4) \cdot 9 \]

\[ V = 540 \]

The volume is 540 ft³.

**Exercises**

Find the volume of each prism.

6.  

7.  

---
**EXAMPLE**

- Find the volume of the cylinder to the nearest tenth. Use 3.14 for \( \pi \).

\[
V = \pi r^2 h
\]

\[
V \approx 3.14 \cdot 3^2 \cdot 4
\]

\[V \approx 113.04\]

The volume is about 113.0 cm\(^3\).

**EXERCISES**

Find the volume of each cylinder to the nearest tenth. Use 3.14 for \( \pi \).

8.  

\[
V = \pi r^2 h
\]

9.  

\[
V = \pi r^2 h
\]

**10-3 Volume of Pyramids and Cones (pp. 490–493)**

**EXAMPLES**

- Find the volume of the pyramid.

\[
V = \frac{1}{3} Bh
\]

\[
V = \frac{1}{3} \cdot (5 \cdot 6) \cdot 7
\]

\[V = 70\]

The volume is 70 m\(^3\).

- Find the volume of the cone to the nearest tenth. Use 3.14 for \( \pi \).

\[
V = \frac{1}{3} \pi r^2 h
\]

\[
V \approx \frac{1}{3} \cdot 3.14 \cdot 4^2 \cdot 9
\]

\[V \approx 150.72\]

The volume is about 150.7 ft\(^3\).

**EXERCISES**

Find the volume of each pyramid.

10.  

11.  

Find the volume of each cone to the nearest tenth. Use 3.14 for \( \pi \).

12.  

13.  

**10-4 Surface Area of Prisms and Cylinders (pp. 498–502)**

**EXAMPLE**

- Find the surface area of the prism.

\[
S = 2B + Ph
\]

\[
S = 2(6) + (10)(4)
\]

\[S = 52 \text{ in}^2\]

**EXERCISES**

Find the surface area of each prism.

14.  

15.  

16.  
EXAMPLE
Find the surface area of the pyramid. Use 3.14 for π.

\[ S = 2\pi r^2 + 2\pi rh \]
\[ = 2\pi(3)^2 + 2\pi(3)(7) \]
\[ = 18\pi + 42\pi \]
\[ = 60\pi \]
\[ \approx 188.4 \text{ m}^2 \]

EXERCISES
Find the surface area of each figure. Use 3.14 for π.

18. 19.

10-6 Spheres (pp. 508–511)

EXAMPLE
Find the volume of a sphere with radius 12 cm. Use 3.14 for π.

\[ V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12^3) \]
\[ = 2304\pi \text{ cm}^3 \approx 7234.6 \text{ cm}^3 \]

EXERCISES
Find the volume of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π.

20. \( r = 6 \text{ in.} \)
21. \( d = 36 \text{ m} \)

10-7 Scaling Three-Dimensional Figures (pp. 512–515)

EXAMPLE
A 4 in. cube is built from small cubes, each 2 in. on an edge. Compare the volumes of the large cube and the small cube.

\[ \frac{\text{vol. of large cube}}{\text{vol. of small cube}} = \frac{4^3 \text{ in}^3}{2^3 \text{ in}^3} = \frac{64 \text{ in}^3}{8 \text{ in}^3} = 8 \]

The volume of the large cube is 8 times that of the small cube.

EXERCISES
A 9 ft cube is built from small cubes, each 3 ft on an edge. Compare the indicated measures of the large cube and the small cube.

22. edge lengths
23. surface areas
24. volumes
Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

1. 

2. 

3. 

Find the volume of each figure to the nearest tenth. Use 3.14 for \( \pi \).

4. 

5. 

6. 

7. 

8. 

9. 

10. Find the surface area of the figure at right.

Find the surface area of each figure to the nearest tenth. Use 3.14 for \( \pi \).

11. 

12. 

13. 

14. a cone with diameter 6 cm and slant height 8 cm

15. a sphere with radius 3 ft

16. The surface area of a rectangular prism is 45 ft\(^2\). What is the surface area of a similar prism that is larger by a scale factor of 3?

17. The volume of a flowerpot is 7.5 cm\(^3\). What is the volume, to the nearest hundredth, of a similar flowerpot that is smaller by a scale factor of \( \frac{1}{2} \)?
Cumulative Assessment, Chapters 1–10

Multiple Choice

1. What is the ratio of gold to silver in the chart below?

<table>
<thead>
<tr>
<th>Precious Metals Company</th>
<th>Supply of Gold and Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td></td>
</tr>
</tbody>
</table>

A. \( \frac{19}{22} \)  
B. \( \frac{13}{19} \)  
C. \( \frac{22}{19} \)  
D. \( \frac{19}{11} \)  

2. The regular price of a bracelet is $96. Serena buys the bracelet on sale at 20% off. How much money does Serena save?

A. $1.92  
B. $9.60  
C. $19.20  
D. $76.80

3. Nationally, there were 217.8 million people age 18 and over and 53.3 million children ages 5 to 17 as of July 1, 2003, according to estimates released by the U.S. Census Bureau. How do you write the number of people age 5 and older in scientific notation?

A. \( 2.711 \times 10^2 \)  
B. \( 2.711 \times 10^6 \)  
C. \( 2.711 \times 10^7 \)  
D. \( 2.711 \times 10^8 \)

4. Which situation describes the graph?

- A. Linda sits on her bike. Linda runs to see the neighbor’s dog. Linda sits and pets the dog.
- C. Carlos runs to answer the phone. Carlos sits and talks on the phone. Carlos walks into another room.
- D. Juan walks to his friend’s house. Juan knocks on the door. Juan leaves his friend’s house.

5. Which expression is NOT equivalent to \( 4 \cdot 4 \cdot 4 \cdot 4 \)?

A. \( \frac{1}{4^5} \)  
B. 20  
C. \( 4^2 \cdot 4^3 \)  
D. 1024

6. A trapezoid has two bases, \( b_1 \) and \( b_2 \), and height, \( h \). For which values of \( b_1 \), \( b_2 \), and \( h \) is the area of a trapezoid equal to 32 in.?

A. \( b_1 = 9 \text{ in.}, b_2 = 7 \text{ in.}, h = 2 \text{ in.} \)  
B. \( b_1 = 5 \text{ in.}, b_2 = 3 \text{ in.}, h = 4 \text{ in.} \)  
C. \( b_1 = 2 \text{ in.}, b_2 = 8 \text{ in.}, h = 4 \text{ in.} \)  
D. \( b_1 = 9 \text{ in.}, b_2 = 7 \text{ in.}, h = 4 \text{ in.} \)
7. The area of a circle is $49\pi$. What is the diameter of the circle?
   - A) 7 units
   - B) 14 units
   - C) 21 units
   - D) 49 units

8. If the value of $a^5$ is positive, then which is true?
   - A) $a$ is positive.
   - B) $a$ is negative.
   - C) $a^5$ is odd.
   - D) $a^5$ is even.

Short Response

14. Plot the points $A(-5, -4)$, $B(1, -2)$, $C(2, 3)$, and $D(-4, 1)$. Use line segments to connect the points in order. Then find the slope of each line segment. What special kind of quadrilateral is $ABCD$? Explain.

15. A cylinder with height 6 in. and diameter 4 in. is filled with water. A cone with height 6 in. and diameter 2 in. is placed inside the cylinder, vertex down, with its base even with the top of the cylinder. Draw a diagram to illustrate the situation described. Then determine how much water is left in the cylinder. Show your work.

16. Rory made a pentagon by cutting two triangles from a square as shown.

What is the area of the pentagon? Show your work.

Extended Response

17. The surface of a geodesic dome is approximately spherical.
   a. A pattern for a geodesic dome that approximates a hemisphere uses 30 triangles with base 8 ft and height 5.63 ft and 75 triangles with base 8 ft and height 7.13 ft. Find the surface area of the dome.
   b. The base of the dome is approximately a circle with diameter 41 ft. Use a hemisphere with this diameter to estimate the surface area of the dome.
   c. Compare your answer from part a with your estimate from part b. Explain the difference.