

Outline of Topics Covered in Higher Level Math 5/6

- Polynomial functions
 - Use Remainder Theorem & Factor Theorem
 - Conjugate roots of polynomials
 - Factor polynomials using graphing calculator
 - Use polynomial long division
 - Use synthetic substitution/division
- Sequences and series
 - Identify sequences: arithmetic vs. geometric vs. other
 - Develop the n th term
 - Use sigma notation
 - Convert between expanded form and sigma notation
 - Sum finite arithmetic sequences
 - Sum finite and infinite geometric series
- Proof by mathematical induction
 - Write inductive proofs: show base case(s) (P_1, P_2, \dots), state inductive assumption (P_k), develop inductive step (P_{k+1}), prove inductive step ($P_k \Rightarrow P_{k+1}$)
 - Form conjectures to be proven by mathematical induction
- The Binomial Theorem
 - Use sigma notation
 - Expand $(a + b)^n$ completely or develop specified terms in the expansion
 - Find the coefficient for a specified term in the expansion
- Trigonometry
 - Area of a triangle in terms of θ
 - Use law of sines (including the ambiguous case) to solve triangles
 - Use law of cosines to solve triangles
 - Represent radian measure graphically on a circle with radius r
 - Convert between degree and radian measure
 - Calculate arc length and sector area
 - Define and sketch the six trigonometric functions
 - Develop, in the context of the unit circle, exact values of sine and cosine for angles that are multiples of $\pi/6$ and $\pi/4$, (reference right triangles)
 - Find solutions for trigonometric equations: principal & secondary, general forms and over a finite interval using the unit circle or sinusoidal graphs
 - Prove & use negative angle, complementary angle & Pythagorean identities
 - Prove trigonometric identities (including sum and difference of two angles, double angle, half angle and compound angle identities)
 - Given a ratio's value (e.g: $\sin \theta = k$), find values of other ratios without finding θ
 - Use trigonometric identities and factorization to transform equations
 - Work with graphs of trigonometric functions: domains, ranges, period length, frequency, amplitude, vertical shift, horizontal (phase) shift
 - Work with $f(x) = a \sin(b(x+c)) + d$ & $f(x) = a \cos(b(x+c)) + d$
 - Work with inverse trigonometric functions ($\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$), their domains, ranges and graphs

- Complex numbers
 - Use vectors as geometric representations for complex numbers and complex number arithmetic
 - Convert between rectangular $(a + bi)$ and polar $(r(\cos \theta + i \sin \theta))$ forms
 - Calculate products and quotients in rectangular or polar forms.
 - Prove DeMoivre's Theorem by mathematical induction
 - Use DeMoivre's Theorem to find powers and roots of complex numbers
 - Develop and use $z^n + \frac{1}{z^n} \equiv 2\cos(n\theta)$ & $z^n - \frac{1}{z^n} \equiv 2i\sin(n\theta)$
- Vectors
 - Work with vectors in 2-space and 3-space
 - Develop 2-space vectors as a linear combination of two base vectors given their directions and magnitudes
 - Work with components of vectors with respect to the standard basis (unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$)
 - Use algebraic and geometric approaches to the following:
 - Find the sum and difference of two vectors
 - Multiplication by a scalar
 - Calculate magnitude of a vector
 - Find components for position and direction vectors
 - Normalize a vector and connect to direction cosines and angles of inclination to the axes
 - Identify parallel and perpendicular vectors
 - Compute the scalar product (dot product) of two vectors and find the angle between two vectors
 - Work with and convert between forms of equations for a line: vector, parametric, Cartesian
 - Distinguish between parallel, intersecting and skew lines
 - Find the point of intersection of lines and the angle formed by intersecting lines
 - Compute the vector product (cross product) of two vectors
 - Geometric interpretation of $|\mathbf{v} \times \mathbf{w}|$
 - Work with and convert between forms of equations for a plane: parametric, normal vector, Cartesian
 - Find intersections of a line with a plane, of two planes and of three planes
 - Find angles formed by a line and a plane, two planes
 - Find distances in three dimensions between points, lines and planes
- Matrices and transformations
 - Use the algebra of matrices: identity matrix, addition, subtraction, multiplication, multiplication by a scalar
 - Calculate determinants of square matrices (up to 3×3) (condition for singularity)
 - Create 2×2 matrices representing linear transformations in 2-space: rotations, reflections, dilations, shears
 - Compose linear transformations
 - Use row operations to solve 3×3 systems and know the conditions for a unique solution, no solution and infinity of solutions.